Graphs of polynomial functions are smooth and continuous.

- **smooth**: graph contains only rounded corners. No sharp corners.
- **continuous**: graph has no breaks and can be drawn without lifting the pencil.
Definition of a Polynomial Function

Let \( n \) be a nonnegative integer and let \( a_n, a_{n-1}, \ldots, a_2, a_1, a_0, \) be real numbers with \( a_n \neq 0 \). The function defined by

\[
f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0
\]

is called a polynomial function of \( x \) of degree \( n \). The number \( a_n \), the coefficient of the variable to the highest power, is called the leading coefficient.
Zeros of Polynomial Functions

If $f$ is a polynomial function, then the values of $x$ for which $f(x)$ is equal to 0 are called the zeros of $f$. These values of $x$ are the roots of the polynomial equation $f(x) = 0$. 
Repeated Zero with Multiplicity $k$

In factoring the equation for the polynomial function $f$, if the same factor $x - r$ occurs $k$ times, but not $k + 1$ times, we call $r$ a repeated zero with multiplicity $k$.

$$f(x) = (x - r)^k.$$
Repeated Zero with Multiplicity $k$

For the polynomial

$$f(x) = 2x(x - 7)^2(x + 5)^3$$

0 is a zero with multiplicity 1
7 is a zero with multiplicity 2
−5 is a zero with multiplicity 3.
Multiplicity and $x$-intercepts

If $r$ is a zero of even multiplicity, then the graph touches the $x$-axis and turn around at $r$.

If $r$ is a zero of odd multiplicity, then the graph crosses the $x$-axis at $r$. 