Chapter 3 Special Section

Karnaugh Maps

Modified by M. L. Malone, Feb 05.

Note to Students

This is *hard to learn on your own*, but the many examples that we'll do in class will clarify it for you… Be patient!

3A.1 Introduction

- Why simplify Boolean functions?
  - Get simpler (and usually faster) digital circuits.
- How to simplify Boolean functions?
  - Use identities (laws)
    - time-consuming
    - error-prone.
  - Use Karnaugh maps (aka K-maps)

3A.1 Introduction

- Maurice Karnaugh
  - In 1953, a telecommunications engineer at Bell Labs.
  - Exploring the "new" field of digital logic and its application to the design of telephone circuits
  - Invented a graphical way of visualizing & then simplifying Boolean expressions.
- Karnaugh map, or K-map, named in his honor.

3A.2 What is a K-map?

- A matrix consisting of rows and columns that represent the output values of a Boolean function.
- Output values in each cell are derived from the *minterms* of a Boolean function.
- *minterm*
  - A ***product*** term that contains all of the function's variables exactly once, either complemented or not complemented.

3A.2 Description of K-maps and Terminology

- The minterms for a function having the inputs *x* and *y* are $\bar{X}\bar{Y}$, $\bar{X}Y$, $X\bar{Y}$, and XY
- The minterms of the Boolean function

$$F(x,y) = XY + \bar{X}Y$$

Minterm	X	Y
$\bar{X}\bar{Y}$	0	0
$\bar{X}Y$	0	1
$X\bar{Y}$	1	0
XY	1	1

3A.2 Description of K-maps and Terminology

- Minterms for a function having three inputs

Minterm	X	Y	Z
$\bar{X}\bar{Y}\bar{Z}$	0	0	0
$\bar{X}\bar{Y}Z$	0	0	1
$\bar{X}Y\bar{Z}$	0	1	0
$\bar{X}YZ$	0	1	1
$X\bar{Y}\bar{Z}$	1	0	0
$X\bar{Y}Z$	1	0	1
$XY\bar{Z}$	1	1	0
XYZ	1	1	1

3A.2 Description of K-maps and Terminology

- K-map has a cell for each minterm.
- Thus, it has a cell for each line for the truth table of a function.
- The truth table for the function *F(x,y) = xy* is shown at the right along with its corresponding K-map.

F(X,Y) = XY

X	Y	XY
0	0	0
0	1	0
1	0	0
1	1	1

X \ Y	0	1
0	0	0
1	0	1

3A.2 Description of K-maps and Terminology

- Another example:
  - Truth table and K-map for the function, *F(x,y) = x + y* .
  - This is equivalent to the OR of all of the minterms that have a value of 1. Thus:

F(X,Y) = X+Y

X	Y	X+Y
0	0	0
0	1	1
1	0	1
1	1	1

X \ Y	0	1
0	0	1
1	1	1

$$F(x,y) = X+Y = \bar{X}Y + X\bar{Y} + XY$$

3A.3 K-map Simplification for Two Variables

- The mintermfunction (last slide) derived from our K-map was not in simplest terms.
- 1st step in simplifying ("reducing") the expression
  - Find adjacent 1s in the K-map that can be collected into groups with sizes 1, 2, 4, 8, … (that is, sizes that are powers of two).
- In our example, we have two such groups.
  - Can you find them?

X \ Y	0	1
0	0	1
1	1	1

3A.3 K-map Simplification for Two Variables

- The number of 1s in each group is a power of two
- OK if groups overlap.

X \ Y	0	1
0	0	1
1	1	1

3A.3 K-map Simplification for 2 variables: RULES

- Groupings contain only 1s
- Diagonal groups not allowed.
- Number of 1s in a group must be a power of 2 even if it contains a single 1.
- ***Groups must be as large as possible.***
- Groups can overlap and wrap around the sides & corners of the K-map.

We've identified the groupings. Now… for EACH GROUP…

- List the minterms vertically.
- Select only the variable(s) for which the values are all alike (all 1s or all 0s) in the set of minterms. (Hint: in a column!)
  - A column of 1s represents the variable
  - A column of 0s represents its complement.
- OR the results of the groups.

X \ Y	0	1
0	0	1
1	1	1

We'll do this in class today!

3A.4 K-map Simplification for *Three* Variables

- We have placed each minterm in the cell that will hold its value.
- Note: Values for the yz combination at the top of the matrix form a pattern that is not a normal binary sequence: 00, 01, 11, 10.

X \ YZ	00	01	11	10
0	$\overline{X}\overline{Y}\overline{Z}$	$\overline{X}\overline{Y}Z$	$\overline{X}YZ$	$\overline{X}Y\overline{Z}$
1	$X\overline{Y}\overline{Z}$	$X\overline{Y}Z$	XYZ	$XY\overline{Z}$

3A.4 K-map Simplification for Three Variables

- First row contains all minterms where *x* is 0.
- First column contains all minterms where *y* and *z* are both zero.

X \ YZ	00	01	11	10
0	$\overline{X}\overline{Y}\overline{Z}$	$\overline{X}\overline{Y}Z$	$\overline{X}YZ$	$\overline{X}Y\overline{Z}$
1	$X\overline{Y}\overline{Z}$	$X\overline{Y}Z$	XYZ	$XY\overline{Z}$

3A.4 K-map Simplification for Three Variables

- Example:

  $F(X,Y,Z) = \overline{X}\overline{Y}Z + \overline{X}YZ + X\overline{Y}Z + XYZ$
- What is the largest group of some-power-of-2 1s ?
  - That is, one 1, two 1s, four 1s, eight 1s, sixteen 1s, etc.

X \ YZ	00	01	11	10
0	0	1	1	0
1	0	1	1	0

3A.4 K-map Simplification for Three Variables

- This grouping tells us that changes in the variables <u>*x* and *y* have no influence upon the value of the function</u> !
- This means that the function,

  $F(X,Y,Z) = \overline{X}\overline{Y}Z + \overline{X}YZ + X\overline{Y}Z + XYZ$

  reduces to *F(x,y,z) = z.*

You could verify this reduction with identities or a truth table.

X \ YZ	00	01	11	10
0	0	1	1	0
1	0	1	1	0

3A.4 K-map Simplification for Three Variables

- Another example!

  $F(X,Y,Z) = \overline{X}\overline{Y}\overline{Z} + \overline{X}\overline{Y}Z + \overline{X}YZ + \overline{X}Y\overline{Z} + X\overline{Y}\overline{Z} + XY\overline{Z}$
- There are (only) two groupings of 1s.
  - Can you find them?

X \ YZ	00	01	11	10
0	1	1	1	1
1	1	0	0	1

3A.4 K-map Simplification for Three Variables

- OK for a group to wrap around the sides of a K-map (see ***pink***).
- What does this ***pink*** group tell us?
  - The values of *x* and *y* are not relevant to the term of the function that is encompassed by the group.
  - What does this tell us about this term of the function?

Minterms in ***pink*** group:

000
010
100
110

only variable same in all is z'

x \ yz	00	01	11	10
0	1	1	1	1
1	1	0	0	1

3A.4 K-map Simplification for Three Variables

- **Green** group?
  - Tells us that only the value of *x* is significant in that group.
- We see that it is complemented in that row, so the other term of the reduced function is x'.
- Our reduced function is F(x,y,z) = x' + z'

Recall that we had six minterms in our original function!

x \ yz	00	01	11	10
0	1	1	1	1
1	1	0	0	1

3A.5 K-map Simplification for Four Variables

- Model can be extended to 16 minterms that are produced by a four-input function.

wx \ yz	00	01	11	10
00	$\overline{W}\overline{X}\overline{Y}\overline{Z}$	$\overline{W}\overline{X}\overline{Y}Z$	$\overline{W}\overline{X}YZ$	$\overline{W}\overline{X}Y\overline{Z}$
01	$\overline{W}X\overline{Y}\overline{Z}$	$\overline{W}X\overline{Y}Z$	$\overline{W}XYZ$	$\overline{W}XY\overline{Z}$
11	$WX\overline{Y}\overline{Z}$	$WX\overline{Y}Z$	$WXYZ$	$WXY\overline{Z}$
10	$W\overline{X}\overline{Y}\overline{Z}$	$W\overline{X}\overline{Y}Z$	$W\overline{X}YZ$	$W\overline{X}Y\overline{Z}$

3A.5 K-map Simplification for Four Variables

- K-map shown below has nonzero minterms indicated

$$F(W,X,Y,Z) = \overline{W}\overline{X}\overline{Y}\overline{Z} + \overline{W}\overline{X}\overline{Y}Z + \overline{W}\overline{X}Y\overline{Z} + \overline{W}XY\overline{Z} + W\overline{X}\overline{Y}\overline{Z} + W\overline{X}\overline{Y}Z + W\overline{X}Y\overline{Z}$$

- Can you identify (only) three groups in this K-map?

Recall that groups can overlap.

wx \ yz	00	01	11	10
00	1	1		1
01				1
11				
10	1	1		1

- Hint:
  - 2 groups of 4
  - 1 group of 1

3A.5 K-map Simplification for Four Variables

- Our three groups consist of:
  - A **purple** group entirely within the K-map at the right.
  - A **pink** group that wraps the top and bottom.
  - A **green** group that spans the corners.
- Thus we have three terms in our final function:

wx \ yz	00	01	11	10
00	1	1		1
01				1
11				
10	1	1		1

3A.5 K-map Simplification for Four Variables

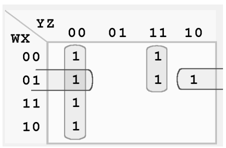
The final function based on these 3 groups!

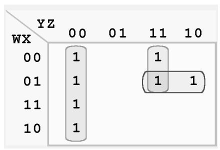
wx \ yz	00	01	11	10
00	1	1		1
01				1
11				
10	1	1		1

$$F(W,X,Y,Z) = \boxed{\overline{X}\overline{Y}} + \boxed{\overline{X}\overline{Z}} + \boxed{\overline{W}Y\overline{Z}}$$

3A.5 K-map Simplification for Four Variables

- More than one solution!
- Depends on how you pick the groups of 1s.
- Keep groups as large as possible!
- See different groupings below
  - Logically equivalent!





More on K-Map Simplification

- What if terms are not all minterms ?
- Ex: F(x,y,z) = x'yz + ***xy***
- Using Boolean laws:

  xy = xyz + xyz' because

  xyz + xyz' = xy(z+z') = xy (1) = xy

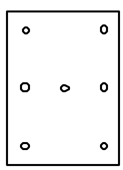
  Thus, F(x,y,z) = x'yz + xy is the same as

  F(x,y,z) = x'yz + ***xyz + xyz'***

  – We now have all minterms in the Boolean function!

3A.6 Don't Care Conditions

- Real circuits don't always require an output for every possible input.
  - Example: Some calculator displays consist of 7-segment LEDs. These LEDs can display 2^7 -1 patterns, but only ten of them are useful.



3A.6 Don't Care Conditions

- Can design circuit so that a particular set of inputs can never happen
  - Such inputs call "don't care"s
  - Helpful in K-map circuit simplification.

3A.6 Don't Care Conditions

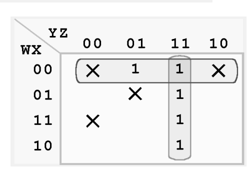
- A <u>don't care</u> condition is identified by *X* in the cell of the minterm(s) for the don't care inputs
- When simplifying, we can include or ignore the Xs when creating our groups.

WX \ YZ	00	01	11	10
00	X	1	1	X
01		X	1	
11	X		1	
10			1	

3A.6 Don't Care Conditions

- In one grouping in the K-map below, we have the function:

$$F(W,X,Y,Z) = \bar{W}\bar{X} + YZ$$



3A.6 Don’t Care Conditions

- A different grouping gives us the function:

$$F(W,X,Y,Z) = \overline{W}Z + YZ$$

WX \ YZ	00	01	11	10
00	×	1	1	×
01		×	1	
11	×		1	
10			1	

31

3A.6 Don’t Care Conditions

- The truth table of $F(W,X,Y,Z) = \overline{W}\,\overline{X} + YZ$ is different from the truth table of $F(W,X,Y,Z) = \overline{W}Z + YZ$
- BUT…the values for which they differ are the inputs for which we have don’t care conditions.

WX \ YZ	00	01	11	10
00	×	1	1	×
01		×	1	
11	×		1	
10			1	

WX \ YZ	00	01	11	10
00	×	1	1	×
01		×	1	
11	×		1	
10			1	

32

3A Conclusion re K-Maps

- Provide an easy graphical method of simplifying Boolean expressions.
- Consist of the outputs of the minterms of a Boolean function.
- We discussed 2 - 3- and 4-input K-maps
  - Method can be extended to any number of inputs.

33

3A Conclusion re K-Maps

Review: Rules for K-map simplification:

- Groupings contain only 1s
- Diagonal groups not allowed.
- Number of 1s in a group must be a power of 2 even if it contains a single 1.
- Groups must be as large as possible.
- Groups can overlap and wrap around the sides & corners of the K-map.

34